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Mortality Swaps and Tax Arbitrage in the Canadian Insurance and Annuity Markets

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Abstract

The authors analyze a tax arbitrage opportunity that results from engaging in two seemingly counterintuitive transactions in the Canadian insurance market. Specifically, if an individual acquires a fixed immediate life annuity and then uses the periodic annuity income to fund a term-to-life insurance policy, these two transactions, which the authors refer to as a "mortality swap," will generate a payoff pattern that is risk-free. In other words, a mortality swap replicates a risk-free security, albeit one with a stochastic liquidation date. The authors show theoretically that the rate of return on a mortality swap is equal to the risk-free rate on a *before-tax* basis, but exceeds it on an *after-tax* basis. This is confirmed by the results of the empirical test, which uses observed annuity and insurance quotes that already reflected adverse-selection and transaction costs. The authors also observe that the older an individual is and/or the higher his/her marginal tax rate is, the more he/she stands to gain from this tax arbitrage.

This advantageous investment opportunity exists because of the arguably lenient method that Canadian authorities use to tax annuity income. The authors provide two major reasons that this method leads to an arbitrage opportunity. The authors then compare this method to that under the U.S. tax rules and show that the U.S. method renders tax arbitrage very unlikely.

The demand for life insurance and annuities is usually attributed to risk aversion, consumption smoothing, and the desire for household protection. The authors' findings provide an arbitrage-based reason for their demand. As a natural by-product, the authors' research contains policy implications for the optimal taxation of annuities and insurance policies.

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INTRODUCTION

The essence of arbitrage-free pricing is that assets with identical payoff patterns should have the same price. The equality of prices and, by extension, rates of return hold true if markets are sufficiently free of frictions. In such markets, investors will be indifferent among the choices of assets with similar payoffs.

In practice, however, market frictions such as transaction costs, income taxes, and asymmetry of information can significantly distort investment decisions by causing one investment alternative to be less costly—thus providing a higher rate of return—than others. Previous research has investigated this possibility, both theoretically and empirically. [See, for example, Jarrow and O'Hara (1989) and Kamara and Miller (1995).]

In this article, the authors document a return discrepancy between the Canadian insurance and fixed-income markets. The authors show that by engaging in seemingly counterintuitive transactions involving two insurance products, one can create a risk-free portfolio whose after-tax return is greater than that of available risk-free securities. The two insurance products in question are (1) a standard term-to-100 life insurance policy and (2) a single-premium fixed immediate life annuity with no guarantee period.

Consider an individual who invests \$100,000 in a fixed immediate life annuity and then uses part of the periodic income from the annuity to pay the premium on a life insurance policy whose death benefit is also \$100,000. This "back-to-back" transaction, which shall henceforth be referred to as a "mortality swap," will create a constant periodic flow of income and will return the original \$100,000 upon the death of the policyowner. This payoff pattern is similar to that of a risk-free investment such as a bank deposit whose principal is redeemed (by the individual's estate) at the time of death.[1]

In a frictionless market, the individual would be indifferent between a mortality swap and a bank deposit. However, the authors show that in the presence of Canadian personal income taxes, a mortality swap yields a considerably higher *after-tax* rate of return than the risk-free rate. This discrepancy is due to the (arguably) lenient methods by which Canadian authorities tax "prescribed" annuity income.[2] The benefit of this lenient taxation cannot be captured by investing in an annuity by itself because of the mortality risk of its returns. However, by combining an annuity and an insurance policy into a mortality swap, the mortality risk of the annuity return is offset by

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[1] The payoff of a mortality swap is risk-free in the sense that there is no variation in it, given that the insurance company does not default. In the same sense, a bank deposit is risk-free except for the bank's default risk. Therefore, the authors' comparison of the two is valid, especially for insurance companies and banks of the highest credit rating.

[2] In Canada, if an individual purchases a life annuity with assets from his/her registered retirement savings plan, the annuity income will be wholly taxable. However, if the annuity is purchased with assets outside a registered plan, only a portion of its income will be taxable. How that portion is determined depends on whether these "non-registered" life annuities qualify for the "prescribed" status. If they do, then the taxable portion remains constant throughout the annuitant's life. In practice, most life annuities in Canada qualify for the prescribed status.

the mortality risk of the insurance return, and the benefit of the lenient tax treatment is captured.

Under the standard arbitrage argument, one can therefore create "arbitrage" profits by borrowing money to fund a mortality swap, thus creating a portfolio with zero initial cost and a positive payoff with probability one. However, this profitable opportunity is not an arbitrage opportunity in the traditional sense because it is not scalable, since individuals are limited by the amount of insurance that they can purchase. In addition, it is not entirely riskless, as tax laws might change without providing a "grandparent" clause. Therefore, a more appropriate way to look at this strategy is from the point of view of an investor who is deciding between a risk-free security and a mortality swap. Based on this point of view, the authors define "arbitrage" for their context as a strategy in which a mortality swap provides a return superior to the risk-free rate. It is under this weaker definition that the authors will use the term "arbitrage" throughout the article.

The authors' research sheds light on the interaction between the insurance and financial markets vis-á-vis the proper taxation of these products. While the practice of back-to-back transactions has been around for a while (mainly among wealthy individuals), there has not been a rigorous study of the magnitude of, or the underlying reasons for, its benefits.[3]

From a micro-economic perspective, the authors' study provides a tax-driven explanation for the demand for insurance and annuities. This explanation differs from, but does not contradict, the classical life-cycle smoothing (risk-aversion) argument proposed in Yaari (1965), Fischer (1973), Campbell (1980), Karni and Zilcha (1986), and Lewis (1989). In addition, while life insurance is extensively studied and widely understood [see the classic book by Black and Skipper (1999)], immediate life annuities are still a growing area [see the work by Poterba (1997); Mitchel, Poterba, Warshawsky, and Brown (2000); Brown, Mitchel, Poterba, and Warshawsky (1999); and Milevsky (2001) for some possible explanations]. This article contributes to the annuity literature and also links it to the concept of arbitrage.

Although the authors' main analysis is done under the Canadian tax rules, the results have implications on other tax jurisdictions as well. For example, under present U.S. tax regulations, life annuities are taxed using a slightly different scheme that results in a higher effective tax rate. The authors explain why tax arbitrage is unlikely under this scheme. Recently, however, Brown, Mitchel, Poterba, and Warshawsky (1999) proposed an alternative scheme that is similar to the Canadian scheme, while preserving the higher effective tax rate. The authors also discuss the possibility of tax arbitrage under this proposed scheme.

The article is organized as follows: the next section analyzes a mortality swap in a frictionless market. In "Markets With Personal Income Taxes," the authors introduce the Canadian personal income taxation into their model and identify the reasons for the existence of an arbitrage opportunity. The authors then present a numerical analysis

[3] Back-to-back transactions are sometimes referred to in the industry as "insured annuities." In practice, insured annuities may not be exactly in the same form or involve the same insurance policies and annuities as the authors describe.

using annuity and insurance quotes from Canadian insurance companies. In addition, the authors discuss the effect of the U.S. personal income taxation. Then, the fourth section examines the effects of other market frictions such as adverse-selection costs and transaction costs that may prevent individuals from taking full advantage of this arbitrage opportunity. The fifth section concludes the article. Proofs and description of the data used in the authors' numerical analysis are contained in the appendices.

FRICTIONLESS MARKETS

The authors start the analysis of mortality swaps by showing that a tax arbitrage opportunity does not exist in a "frictionless" market, which the authors define as one with no taxes, transaction costs, or adverse-selection concerns. In addition, the authors assume for now a flat term structure of interest rates at the (continuously compounded) constant per-annum rate of r. Later in the article, the authors will discuss the case of a non-flat term structure and stochastic interest rates. The basic intuition for the existence of arbitrage is the same under either interest-rate assumption.

Life Annuities in Frictionless Markets

Consider an immediate life annuity that guarantees a lifetime payment of $1 per annum (in continuous time), starting immediately. Under the assumed market structure, the actuarially fair price of this annuity for an individual aged x is given by:

$$a_x(r) = \int_0^\infty e^{-rs}\ {}_sp_x ds, \tag{1}$$

where e^{-rs} is the discount factor applicable to a $1 payment at time s, and ${}_sp_x$ is the conditional probability that an x-year-old individual will survive for another s years (i.e., to age $x + s$).[4] The pricing relationship in Equation (1) is indeed the *actuarial principle of equivalence*, which equates the present value of the expected benefits to the initial price.[5]

As a corollary, a $1 investment in a life annuity at age x will entitle the annuitant to lifetime annual payments in the amount of $1/a_x(r)$ per year.

Clearly, a mortality risk is associated with the effective rate of return that an individual will earn on an annuity. The longer he/she lives, the more payments he/she will receive and thus the higher is the effective return.

Life Insurance in Frictionless Markets

Consider a life insurance policy that pays $1 at the time of death and requires a constant premium to be paid every year (in continuous time). One can think of this policy

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[4] As a special case, if lifetime is exponentially distributed, then the annuity price, $a_x(r)$, is equal to $1/(\lambda + r)$, where λ is the (constant) instantaneous force of mortality. The authors note, however, that exponential distributions are not consistent with empirical mortality patterns. The authors mention it here because it has a bearing on the argument that will be developed later in the article.

[5] See Bowers et al. (1986) for a description of the underlying assumptions and the law of large numbers used in actuarial pricing.

as a term-to-life insurance policy. In a world without adverse selection, insurance companies use the same mortality table to price both annuities and insurance policies. Given this assumption, the actuarially fair annual premium for an individual aged x is given by:

$$i_x(r) = \frac{\int_0^\infty e^{-rs}\,{}_sp_x \lambda_{x+s} ds}{\int_0^\infty e^{-rs}\,{}_sp_x ds} = \frac{\int_0^\infty e^{-rs}\,{}_sp_x \lambda_{x+s} ds}{a_x(r)}, \tag{2}$$

where $\lambda_{x+s} > 0$ is the instantaneous force of mortality, and the second equality comes from Equation (1).[6]

As in the case of annuities, a mortality risk is associated with the effective rate of return on an insurance policy. The longer a policyholder lives, the more premium payments he/she makes and thus the lower the effective return.

Mortality Swaps in Frictionless Markets

Suppose an x-year-old individual uses \$1 to purchase a life annuity. According to the above corollary, his or her lifetime annuity income is constant at $\$1/a_x(r)$ per year. Suppose the individual then buys a life insurance policy whose death benefit is also \$1. The premium for this policy is constant at $\$i_x(r)$ per year. By constructing this mortality swap, the individual has created a cash flow stream of $\$1/a_x(r) - i_x(r)$ as long as he or she is alive and \$1 at the time of death. This cash flow pattern is similar to that of a \$1 investment in a risk-free instrument such as a bank deposit that pays interest at a percentage rate of $1/a_x(r) - \$i_x(r)$ every year and whose principal is redeemed at par (by the individual's estate) at the time of death. Therefore, by holding both an annuity and an insurance policy, the individual removes the mortality risk in the effective rate of return on the combined position.

In a frictionless market, the rate of return on a mortality swap is equal to the risk-free rate; i.e.,

$$\frac{1}{a_x(r)} - i_x(r) = r. \tag{3}$$

Equation (3) is in fact a well-known actuarial identity whose proof can be found in Bowers et al. (1986).[7] The authors state this formally as a theorem.

Theorem 1 In a frictionless market—i.e. no taxes, transaction costs, or adverse selection—with a flat term structure of interest rates, a mortality swap will not admit arbitrage. In other words, an investment in a mortality swap has the same lifetime and death payoffs as those of a risk-free investment such as a bank deposit.

Proof As stated.

In the special case of exponential mortality, the yearly annuity income on a \$1 investment is $\$(\lambda + r)$, and the insurance premium on a \$1 policy is $\$\lambda$, resulting in a cash flow stream of $\$r$ per year from a mortality swap.

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[6] Again, as a special case, if lifetime is exponentially distributed, then Equation (2) reduces to $i_x(r) = \lambda$.

[7] The proof by Bowers et al. was done in discrete time. A continuous-time proof is available from the authors on request.

MARKETS WITH PERSONAL INCOME TAXES

In this section, the authors relax the frictionless assumptions by introducing personal income taxation into the framework. Specifically, the authors assume that individuals are subject to the Canadian income tax rules. The authors want to examine the effect of the rules on the payoffs of annuities and insurance policies. The authors will then compare a mortality swap to a risk-free investment. The authors' main result is that when income taxation on annuities and insurance policies is not implemented properly, a tax arbitrage opportunity will exist. The authors then discuss the U.S. tax rules and the possibility of tax arbitrage under them.

Life Annuities in the Canadian Markets

The Canadian Income Tax Act (hereafter referred to as the "Act") considers income from a prescribed life annuity as consisting of two parts. The first part is the return *of* capital (principal) and is, therefore, not taxed. The second part is considered interest (or return *on* capital) and is fully taxable. The percentage of each annuity payment that will be taxed (hereafter referred to as the "taxable portion," and denoted by ρ) is specified by the Act based on the price of the annuity, the income from the annuity, and the age of the purchaser, x (which determines his or her expected remaining lifetime), according to the following formula:[8]

$$\rho_x = 1 - \frac{1}{E[T_x^g]\frac{1}{a_x(r)}} = 1 - \frac{a_x(r)}{E[T_x^g]}, \tag{4}$$

where $1/a_x(r)$ is, as before, the yearly annuity income that an x-year-old individual will receive from a \$1 investment. The central variable $E[T_x^g]$ is the individual's expected remaining lifetime based on the mortality table specified by the Act, which currently is *The Society of Actuaries 1971 Individual Annuity Mortality (IAM) Table*. The second term on the right side of either equality represents the portion of each annuity income that is considered to be the return *of* capital. Once determined, the taxable portion, ρ_x, will remain the same throughout the annuitant's life, even if he/she lives beyond his or her expected remaining lifetime, $E[T_x^g]$ years.

The after-tax annual income from a \$1 annuity investment is therefore constant at:

$$\frac{1}{a_x(r)} - \frac{\tau\rho_x}{a_x(r)},$$

where τ is the annuitant's marginal tax rate, which is assumed to be constant throughout his or her remaining lifetime.[9]

[8] See Reg. 300 (1.1) of the Canadian *Income Tax Act* (2000). The U.S. Internal Revenue Service also uses this formula. However, the implementation is different. The authors will discuss the U.S. case later in the article.

[9] As will be shown, the existence (although not the magnitude) of the tax arbitrage is independent of personal income-tax rates. Therefore, this assumption will not affect the authors' results but will facilitate the exposition.

Three things are worth noting on the effect of taxes on annuity income, all of which have implications on the possibility of tax arbitrage. First, the taxable portion, ρ_x, remains constant throughout the annuitant's life. This means that annuity income is never fully taxed, even after the principal is considered to be completely returned. Second, the choice of the significantly outdated 1971 individual annuitant mortality (IAM) table causes $E[T_x^g]$ to be shorter than present life expectancies, which are used by insurance companies to price annuities. This causes the taxable portion of annuity income to be lower than it should be. Finally, given the annuity price, $a_x(r)$, the value of ρ_x depends only on the expected value or the first moment of the distribution of future lifetime, $E[T_x^g]$, and not on any higher moments. These three issues will be discussed in detail shortly.

Life Insurance in the Canadian Markets

Under the Canadian tax rules, life insurance policies that are issued after December 1, 1982, are classified into two classes depending on their tax treatment—exempt and non-exempt.[10] The distinction is based on the extent of the investment (or savings) component of the policies. The lower this component is, the greater is the chance that the policies will be exempt. The policyholder is required by the Act to have his or her policy tested on each of its anniversaries. For policies that pass this test, taxation is deferred until disposition occurs. Because the death of policyholders does not constitute disposition, death benefits from exempt policies are not taxable.

Generally, term-life policies (including the one used in the authors' construction of a mortality swap) qualify as exempt policies. Therefore, their death benefits are usually not taxable. As for their premiums, they are not tax-deductible. Hence, there is no effect of taxes on the amounts of premium and death benefit. The annual premium of $\$i_x(r)$ results in an after-tax payment of \$1 at the time of death.

Mortality Swaps in the Canadian Markets

The authors again compare a \$1 investment in a mortality swap to a \$1 bank deposit. The after-tax payoffs from the two investment choices to an individual who is currently x-years-old are:

Investment's After-Tax Payoff	State of Nature	
	While Alive	At Death
Mortality Swap	$\$\frac{1}{a_x(r)} - \$\frac{\tau\rho_x}{a_x(r)} - \$i_x(r)$ per year	\$1
Bank Deposit	\$ $(1 - \tau)\, r$ per year	\$1

[10] See Regulation 306 of the Canadian *Income Tax Act*. For a practitioner's discussion of this rule, see, for example, Lengvari and Joshua (1995).

Because of the constant nature of its after-tax payoff, this mortality swap can still be thought of as a synthetic bank deposit on an after-tax basis. Because the costs and the final payoffs under both investment alternatives are the same, tax arbitrage is possible if the mortality swap provides higher intermediate after-tax payoffs than that of the bank deposit; i.e.,

$$\frac{1}{a_x(r)} - \frac{\tau\rho_x}{a_x(r)} - i_x(r) > (1-\tau)r. \tag{5}$$

Intuitively, because their before-tax payoffs are the same [as per Equation (3)], tax arbitrage will occur if the tax payment under the swap is lower than under the bank deposit; i.e.,

$$\frac{\tau\rho_x}{a_x(r)} < \tau r,$$

which leads to the arbitrage condition:

$$\rho < ra_x(r)\,. \tag{6}$$

Therefore, in the current framework, the existence of tax arbitrage depends on the interaction among the taxable portion, the interest rate, and the price of the annuity. On the other hand, it does not depend on the individual's marginal tax rate, τ, as long as τ is greater than zero. However, from Equation (5), the higher the individual's marginal tax rate, τ, the more he/she stands to gain from this mortality swap.

Before discussing the reasons this tax arbitrage does exist in practice, the authors first present the results of their numerical analysis of the real-life annuity and insurance quotes.

A Numerical Study

Table 1 reports annual insurance premiums and annuity income for female individuals of various ages. The insurance premiums are based on a term-to-100 life insurance policy with a death benefit of $100,000, while the annuity income is based on a $100,000 investment in an immediate life annuity.[11] Based on the swap's after-tax payoffs for various marginal tax rates, the authors calculate "grossed-up returns." These are the equivalent pre-tax rates of return that an individual would have to obtain on a fixed-income investment of the same risk in order to end up with the same after-tax returns.

For example, consider a 60-year-old female who is in good health and whose marginal tax rate is 50 percent.[12] She would pay $1,820 per annum (in monthly installments) under the insurance policy. The average of the best five annuity quotes based

[11] The insurance and annuity quotes are as of July 1, 1999. Please see Appendix A for a detailed discussion about the quotes.

[12] In Canada, 50 percent is approximately the highest combined federal and provincial marginal tax rate. In the province of Ontario, it takes effect beyond an annual taxable income of $74,000 (as of September 2000).

on a \$100,000 purchase would provide a 60-year-old female with \$7,397 per annum (in monthly installments), of which \$3,272 is taxable. At the 50 percent marginal tax rate, this would result in an annual after-tax annuity income of \$5,761, and a net cash flow from the swap of \$3,941 per annum for life. Therefore, the after-tax rate of return on the swap is 3.94 percent p.a. Based on her 50 percent tax bracket, this translates to an equivalent pre-tax rate of return of 7.88 percent p.a. This equivalent rate was higher than to the yield on a bank deposit or a guaranteed investment certificate (GIC) which, at the time the authors obtained the quotes (July 1, 1999), was around 5 percent p.a.[13] As a result, this individual would have an arbitrage opportunity.

The existence of this arbitrage opportunity can also be ascertained by checking whether the arbitrage condition in Inequality (6) holds. In this case, a_{60}, which is the annuity price per \$1 of yearly income, is equal to \$100,000/7,397 = \$13.519. The taxable portion, ρ_{60}, is equal to \$3,272/7,397 = 0.44234, while the risk-free rate is 5 percent p.a. Hence, ρ_{60} is less than ra_{60}, and tax arbitrage exists.

TABLE 1
Annual Insurance Premiums, Annuity Income, and Equivalent Pre-Tax Rates of Return on a Mortality Swap for Females of Various Ages and Marginal Tax Rates

Age	Annual Insurance Premium	Annual Annuity Payment	Annual Taxable Income	Equivalent Pre-Tax Return		
				τ = 25%	τ = 40%	τ = 50%
50	\$ 890	\$ 6,612	\$ 3,603	6.43%	7.13%	7.84%
55	\$ 1,276	\$ 6,917	\$ 3,421	6.38%	7.12%	7.86%
60	\$ 1,820	\$ 7,397	\$ 3,272	6.35%	7.11%	7.88%
65	\$ 2,566	\$ 8,112	\$ 3,137	6.35%	7.15%	7.96%
70	\$ 3,566	\$ 9,357	\$ 3,136	6.68%	7.56%	8.45%
75	\$ 4,919	\$ 10,812	\$ 2,709	6.95%	8.02%	9.08%
80	\$ 6,654	\$ 13,162	\$ 2,145	7.96%	9.42%	10.87%

To get a measure of the sensitivity of the above results to the insurance premium, the authors perturb the numbers and then calculate the equivalent rates of return again. For example, suppose the 60-year-old female does not qualify for the "cheap insurance," and has to pay 50 percent more (\$2,730 instead of \$1,820) for the term-to-100 policy. The grossed-up return from the mortality swap would be 6.06 percent (in contrast to 7.88 percent). In the other direction, if the life insurance price were reduced by 25 percent (\$1,365 instead of \$1,820), the grossed-up return from the mortality swap would increase to 8.79 percent. Clearly, the state of health (i.e., the ability to pass a medical examination) will be crucial to the possibility and benefit of tax

13 Note that in this article, the authors assume a flat deterministic term structure. In practice, the rates for bank deposits or GICs can change from year to year. As a result, a more appropriate rate to be compared to in the authors' framework may be the yield of a corporate (i.e., bank's) bond whose maturity is approximately the same as the individual's estimated remaining lifetime. Even with this new benchmark, which was around 6 percent to 6.5 percent as of July 1, 1999, the swap's 7.88 percent return still far exceeded it.

arbitrage.

A few stylized facts are evident from Table 1. First, all the grossed-up rates of return are higher than the current deposit rates (or bond yields). Second, the higher the individual's marginal tax rate, the higher is the grossed-up return from a mortality swap. Therefore, everything else being equal, a higher tax rate implies (1) a higher likelihood that an individual will have an arbitrage opportunity from a mortality swap and (2) a larger magnitude of gain from it. Third, the grossed-up rates of return on the mortality swap (roughly) increase with age. The authors conjecture that this is most likely driven by the insurance loads. It appears that the older an individual is, the more likely is the arbitrage opportunity and the more gain he/she can obtain from it. In fact, for certain combinations of age and tax rate, the swap's returns are approximately twice as high as the risk-free rate.[14]

The Reasons for Tax Arbitrage

The arbitrage condition in Inequality (6) suggests that tax arbitrage will exist if the taxable portion is less than the product of the interest rate and the annuity price; i.e.,

$$\rho_x < r a_x(r) .$$

For a given level of interest rate, r, the higher the annuity price, $a_x(r)$, the more likely is tax arbitrage. This may seem counterintuitive considering the appearance of Inequality (5). However, the authors know from Theorem 1 that the difference between $1/a_x(r)$ and $i_x(r)$ is always equal to r, regardless of the value of $a_x(r)$.[15] That is, the effect of an increase (decrease) in the annuity price is offset by a decrease (increase) in the insurance premium. Therefore, the only effect of a higher $a_x(r)$ is on the term $\tau\rho_x/a_x(r)$, which can be shown to be a decreasing function of $a_x(r)$. Intuitively, an increase in the annuity price is beneficial because it reduces the tax payments.

As a result, given a level of r, the discussion on the possibility of tax arbitrage can be done in terms of the annuity price, $a_x(r)$. For this purpose, the authors rewrite Inequality (6) by using the definition of ρ_x as follows:

$$a_x(r) > \frac{1}{\frac{1}{E[T_x^g]} + r}. \tag{7}$$

In this form, the condition explicitly states that tax arbitrage will occur when the annuity price is higher than a certain threshold. The authors note that this threshold,

$$\frac{1}{\frac{1}{E[T_x^g]} + r},$$

[14] In addition, the swap's returns are higher if, for example, a couple create it by using a last-to-die insurance policy (i.e., a policy issued on two lives under which premiums are required, and death benefits are not paid, until both of them have died) and a joint-and-100%-survivor annuity (i.e., an annuity issued on two lives under which payments continue in whole until both have died).

[15] The authors assume that return on the money invested by insurance companies is not taxable, which is a close approximation to the case in practice.

can be thought of as the annuity price under an exponential mortality distribution with a life expectancy of $E[T_x^g]$ years. This follows from the fact that under exponential distributions, $a_x(r) = 1/(\lambda + r)$ and λ is the reciprocal of life expectancy.

The intuition for this threshold is crucial for the authors' discussion on the reasons that the arbitrage condition such as Inequality (7) can be satisfied. As noted earlier, given the annuity price, $a_x(r)$, the value of ρ_x depends only on the expected value or the first moment of the distribution of lifetime, $E[T_x^g]$. This is *as if* Canada Customs and Revenue Agency (CCRA) *were* assuming that human mortality could be completely specified by its first moment. This would be correct if and only if population mortality were exponentially distributed, since one would need only to know the force of mortality, λ, to generate an exponential mortality distribution. It follows by extension that CCRA inadvertently assumes that annuities are priced according to an exponential distribution whose first moment is equal to $E[T_x^g]$ years. In other words, by stipulating $E[T_x^g]$ and the formula for ρ_x, CCRA indirectly specifies a threshold for the price of an annuity. Tax arbitrage will not exist *if and only if* the annuity price is less than or equal to that threshold. The authors state it as a theorem.

Theorem 2 In a frictionless market with taxation and a flat term structure of interest rates, there is an upper bound for the annuity price for an x-year-old individual, below which a mortality swap will not admit tax arbitrage. That upper bound is the price of the annuity if it is priced based on an exponential distribution with a life expectancy of $E[T_x^g]$ years.

Proof As stated and based on Inequality (7) and the properties of an exponential distribution whose first moment is $E[T_x^g]$ years.

Theorem 2 provides an insight into two major reasons the annuity price quoted by insurance companies can be higher than the threshold and thus tax arbitrage can occur. The first and more obvious reason follows directly from the theorem. It involves the case in which CCRA specifies a life expectancy, $E[T_x^g]$, that is too short relative to that assumed by the insurance companies, $E[T_x^c]$. To isolate its effect, the authors assume for now that insurance companies use an exponential distribution. In this case, the quoted annuity price will be higher than the threshold.

Theorem 3 ***First Reason for Tax Arbitrage:*** Assume a frictionless market with taxation and a flat term structure of interest rates. If insurance companies use an exponential mortality distribution in their pricing, then tax arbitrage will exist if $E[T_x^g] > E[T_x^c]$.

Proof Under the exponential assumption, annuity prices are an increasing function of life expectancies. Therefore, if $E[T_x^g] < E[T_x^c]$, then the price obtained with a life expectancy of $E[T_x^c]$ will be higher than that obtained with $E[T_x^g]$, which is the upper bound for tax arbitrage.

$E[T_x^g]$ will be smaller than $E[T_x^c]$ if CCRA uses a mortality table that is not up-to-date and, therefore, does not reflect recent improvements in life expectancies. As stated earlier, for individual annuitants, the Act is currently using the 1971 individual annuitant mortality (IAM) table, as compiled by the Society of Actuaries. This 30-year-old IAM table is significantly out of date, as evidenced by Table 2, in which it is compared to two more recent IAM tables for 55-year-old individuals. Life expectancies are lower in the 1971 table than in the 1996 table by approximately 3.5 years for females and 4.5 years for males. The probabilities of survival to various ages in the 1971 table are also lower, especially at the tail of the distribution.

TABLE 2
A Comparison of Probabilities of Survival and Life Expectancies of a 55-Year-Old Individual Under Three Annuity Mortality Tables

Probability of Survival to Age	1971 IAM Table		1983 IAM Table		1996 IAM Table	
	Female	Male	Female	Male	Female	Male
55	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
60	97.6%	95.2%	98.2%	96.6%	98.5%	97.4%
65	93.8%	88.6%	95.6%	91.9%	96.2%	93.7%
70	88.9%	79.9%	91.4%	84.8%	92.6%	88.0%
75	81.2%	68.2%	84.9%	74.2%	89.9%	79.1%
80	68.9%	53.0%	74.5%	59.6%	77.5%	66.3%
85	50.4%	35.3%	58.6%	41.5%	62.8%	49.6%
90	28.1%	18.1%	37.9%	23.4%	42.7%	31.3%
95	10.3%	5.6%	18.1%	10.0%	22.1%	15.4%
100	2.6%	0.7%	5.9%	2.89%	8.2%	5.6%
Life Expectancy at Age 55 (yrs.)	31.76	27.49	34.03	29.70	35.22	31.93

As an example of the existence of tax arbitrage under the premise of Theorem 3, suppose that insurance companies price their annuities and insurance policies based on an exponential distribution with a mean lifetime, $E[T_x^c]$, equal to that specified in the 1996 table. Suppose also that the current risk-free rate is 7 percent p.a. In this case, from Table 2, $E[T_x^c]$ of a 55-year-old female is 35.22 years, which means λ is 0.0284. Therefore, the price of the annuity that will pay off $1 per year (in continuous time) to a 55-year-old female is:

$$a_{55}(0.07) = \frac{1}{\lambda + r} = \frac{1}{0.0284 + 0.07} = \$10.1626.$$

However, the arbitrage threshold for the price of this annuity is calculated based on the 1971 table with $E[T_x^g] = 31.76$ years to be:

$$\frac{1}{\frac{1}{31.76} + 0.07} = \$9.8536.$$

Hence, the annuity price quoted by the insurance companies is higher than the threshold, and tax arbitrage will exist.

Intuitively, by using an outdated annuity table, CCRA is inadvertently enhancing the after-tax payoff of the annuity. This is because the outdated life expectancies will cause ρ_x to be too small, and thus a larger portion of annuity income to be exempted from taxes. Suppose a 55-year-old female individual with a 40 percent marginal tax rate invests \$1 in the above annuity. The taxable portion of her annuity income based on the 1971 table is:

$$\rho_{55} = 1 - \frac{10.1626}{31.76} = 0.6800,$$

resulting in an after-tax income of:

$$\frac{1 - 0.4(0.6800)}{10.1626} = \$0.0716.$$

However, if CCRA were to use the 1996 table in specifying $E[T_x^g]$, ρ_{55} would be 0.7115, and the after-tax annual income would be \$0.0704, a difference of \$0.0012 per year (or 0.12 percent p.a. of the annuity principal, \$1).

The above example shows that tax arbitrage will exist if insurance companies use an exponential distribution in their pricing, and $E[T_x^g] < E[T_x^c]$. However, observed mortality patterns are not exponentially distributed. Hence, insurance companies do not use exponential distributions. Without specifying the exact distribution used, the authors cannot state with certainty that the annuity price will be higher than the threshold (and thus tax arbitrage will exist). However, the authors can claim that tax arbitrage will exist whenever insurance companies price their products using a certain class of mortality distributions. More precisely, the authors argue that if actuaries use a mortality distribution from this class, tax arbitrage can exist *even without* $E[T_x^g]$ being less than $E[T_x^c]$. This is indeed the second reason that annuity prices can be higher than their thresholds.

To prove this argument, the authors isolate the effect of a different mortality distribution by assuming for now that $E[T_x^g]$ is the same as $E[T_x^c]$. Under this assumption, Theorem 2 states that if an exponential distribution is used to price annuities, then tax arbitrage will not exist. Consider, however, an alternative mortality distribution for an x-year-old individual with the following properties:[16]

- There exists some age y; $y > x$, such that (a) the probability of survival up to *any* age before y under the distribution is higher than its exponential counterpart; and (b) the probability of survival up to *any* age beyond y under the distribution is lower than its exponential counterpart.

This property partitions the time horizons into two parts according to the difference in the probabilities of survival between the two distributions. The authors call the class of distributions that satisfy this property the "equal-mean, thinner-tail" (EMTT) distributions.[17] The authors argue that annuity prices under this class of distributions will be higher than the exponential prices, and thus arbitrage exists. The authors state this as a theorem.

[16] By definition, a mortality distribution implies probabilities of survival that are non-increasing with time horizon.

[17] This "equal-mean, thinner-tail" property may appear to be similar to one of the definitions of "greater riskiness" proposed by Rothschild and Stiglitz (1970). In that article, Rothschild

Theorem 4 *Second Reason for Tax Arbitrage:* Assume a frictionless market with taxation and a level of interest rate that is positive and the same for all terms. Assume also that $E[T_x^g] = E[T_x^c]$. Then, tax arbitrage will exist if insurance companies use a mortality distribution from the "equal-mean, thinner-tail" (EMTT) class in their pricing.

Proof The authors show in Appendix B that annuity prices under the EMTT class of distributions are higher than the exponential prices, given that $E[T_x^g] = E[T_x^c]$. By Inequality (7), this implies a tax-arbitrage opportunity.

Empirical mortality distributions such as the IAM tables complied by the Society of Actuaries belong to this EMTT class. In Table 3, the authors compare the probability that a 55-year-old female will survive to various ages under the 1996 IAM table to that implied by the exponential distribution with the same life expectancy (i.e., $E[T_{55}] = 35.22$years, and $\lambda = 1/35.22 = 0.0284$). The IAM numbers satisfy the above property. That is, the probability of survival to various ages is higher than its exponential counterpart up to (approximately) age 90 and lower than the exponential numbers afterwards.

TABLE 3
Probabilities of Survival of a 55-Year-Old Female Under an Exponential Distribution and Under the 1996 IAM Table

Probability of Survival to Age	Exponential Distribution	1996 IAM Table
55	100.0%	100.0%
60	86.8%	98.5%
65	75.3%	96.2%
70	65.3%	92.6%
75	56.7%	86.9%
80	49.2%	77.5%
85	42.7%	62.8%
90	37.0%	42.7%
95	32.1%	22.1%
100	27.9%	8.2%
Life Expectancy at Age 55 (years)	35.22	35.22

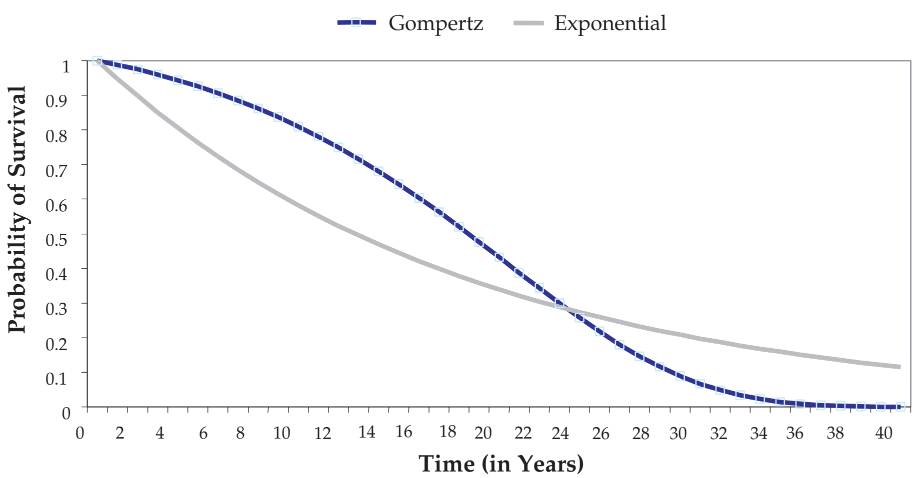
In this case, if annuities are priced based on the 1996 IAM table, then $a_x(r)$ will be higher than its exponential price and tax arbitrage will exist. The intuition for this is as follows. Recall from Equation (1) that the price of an annuity is a weighted sum of

and Stiglitz show that if random variables x and y have the same mean but y has more weight in the tail of its density function than x does, then y is riskier than x in the sense that every person with a concave utility function will prefer x to y. In the present authors' context, the difference in the weights in the tails does not equate with the difference in riskiness as the time of death is not the object of choice.

the present values of all future annuity income, where the weights are the probabilities that the buyer will live to receive those payments. For any given time horizon, the 1996 IAM table assigns a different weight, ${}_{s}p_{x}$, from that assigned by the exponential distribution. If the interest rate were zero, then the two sets of weights would not matter because the weighted sum would turn out to be the same as and equal to the life expectancy. However, when the interest rate is positive, the present value factors cause earlier weights to be more important than later weights. Because earlier weights are higher under the 1996 IAM table than under the exponential distribution, the weighted sum will be greater under the 1996 IAM table.

As another example, one theoretical distribution that has been recognized as providing a realistic description of the pattern of mortality at adult ages is the Gompertz distribution.[18] In Figure 1, the authors provide an illustration of what is meant by the "equal-mean, thinner-tail" class of distributions. The dotted line represents the probability that a person with a remaining life expectancy of 18.5 years will survive to various points in time under the Gompertz distribution. The solid line represents the probability of survival under the exponential distribution with the same life expectancy. The two lines intersect at the time horizon of 24.08 years, beyond which the probability of survival under the Gompertz distribution is below that of the exponential distribution. In other words, the Gompertz distribution has a "thinner-tail" than that of the exponential distribution.

Figure 1
A Comparison of Probabilities of Survival of an Individual Whose Life Expectancy Is 18.5 Years Under a Gompertz Distribution and an Exponential Distribution



As an example of the existence of tax arbitrage under the premise of Theorem 4, the authors calculated the per-annum percentage *difference* between the after-tax return

[18] See, for example, Carriere (1992).

on a mortality swap, and the after-tax return on a risk-free bank deposit, for individuals of various ages, assuming that Gompertz distributions are used to price annuities and insurance policies. The Gompertz distributions were calibrated to the 1996 IAM numbers for those individuals so that, among other things, their life expectancies are the same as in the 1996 IAM tables. As before, the before-tax risk-free return is assumed to be 7 percent p.a. and the marginal tax rate is 40 percent. Table 4 reports the results of the authors' calculations.[19]

Table 4

After-Tax Gains From a Mortality Swap as a Percentage of Investment, Assuming That the Tax Authority Uses Gompertz Distributions with Life Expectancies Based on the 1996 IAM Table

Age	Male	Female
35	0.666%	0.651%
45	0.722%	0.720%
55	0.763%	0.782%
65	0.773%	0.821%
75	0.736%	0.813%
85	0.637%	0.731%
95	0.482%	0.563%

Note: The gains are calculated under an assumption that the risk-free rate is 7% p.a. and the marginal tax rate is 40%.

As can be seen, all differences are positive, consistent with Theorem 4 that arbitrage exists in this case. The magnitude of the differences varies, depending on the age of the individuals. The greatest difference occurs when the buyers are either a 65-year-old male or a 65-year-old female. The differences are lower in magnitude on either side of that age.[20] The authors caution the reader that this "maximum difference" at age 65 may be driven by the mortality function in question, as opposed to the actual taxable portion.

In sum, there are two related reasons that a mortality swap should lead to a tax arbitrage opportunity in the presence of taxation. First, the after-tax payoff from the annuity part of the swap is increased by the fact that an outdated mortality table is specified by CCRA for the purpose of computing the taxable portion. This reason alone would be sufficient to cause tax arbitrage if insurance companies priced their products by an exponential mortality distribution. With a different distribution, this

[19] The details of the authors' calculations are available upon request.

[20] It may appear that there is a "best" time (i.e., age) to purchase a mortality swap. However, the differences in Table 4 are in per-annum percentages, not dollar amounts. Clearly, if a 35-year-old individual constructs a mortality swap, the percentage difference may be lower than what a 55-year-old will get. However, the (expected) length of time over which the difference will be realized is much longer because a 35-year-old has a longer life expectancy than does a 55-year-old.

reason will at least increase the likelihood of tax arbitrage. Second, and more important, the choice of mortality distribution used by insurance companies can cause tax arbitrage, even if CCRA does not use an outdated table. If the employed distribution belongs to the EMTT class, then tax arbitrage will exist. Because it is reasonable to expect actuaries to use a table that is similar in structure to the IAM tables, which themselves belong to the EMTT class, the authors believe that both reasons for tax arbitrage are applicable in practice.

The numerical examples under both reasons show that the second reason is more important than the first. While it is straightforward to see where the difference in after-tax returns come from under the first reason, the second reason warrants further explanation.

When the employed distribution comes from the EMTT class, the existence of tax arbitrage can be traced back to the manner in which prescribed annuities are taxed. To see this, note that in reality, each payment of annuity income consists of four different portions—(1) interest on own principal; (2) return of own principal; (3) a share of principal of annuitants who have already died; and (4) a share of interest on the principal of annuitants who have already died. The relative weights of these four portions will vary through time for two reasons. First, since the annuitant's own principal is gradually returned, earlier annuity payments contain a higher portion of interest on it than later payments do. Second, as more and more annuitants die, the share of principal and interest of the deceased annuitants also increases with time. Therefore, in reality, taxable income from an annuity will vary through time. The variation depends on interest rate and mortality patterns.

However, for tax purposes, each prescribed annuity payment is considered to have only two portions—(1) return of own principal and (2) the rest, which the authors will refer to as the "taxable income." This implies that the Act treats as one portion the interest on own principal and the share of principal and interest of the deceased annuitants. In addition, the Act does not take into account the variation in the relative weights of the two portions. Rather, by specifying ρ_x that does not vary through time, the Act, in effect, dictates that the total taxable income earned *during the annuitant's life expectancy*, $E[T_x^g]$ years,[21] should be assigned to each tax year within that period on a straight-line basis. To see this, recall that a \$1 investment in an annuity will yield annual before-tax income of $\$1/a_x(r)$, of which $\$r_x / a_x(r)$ is taxable. From the definition of r_x in Equation (4),

$$\frac{\rho_x}{a_x(r)} = \frac{E[T_x^g]\dfrac{1}{a_x(r)} - 1}{E[T_x^g]}.$$

The denominator on the right side is indeed the total income that the annuitant will receive over his or her life expectancy minus the initial investment. In other words, the numerator is the total *taxable income* to be earned over $E[T_x^g]$ years. It is then

[21] Note that in order to isolate the effect of "equal-mean, thinner-tail" distributions, the authors continue to assume that $E[T_x^g] = E[T_x^c]$.

divided by the life expectancy to arrive at the taxable income for each of the $E[T_x^g]$ years. Note that this determination has no connection at all to the actual composition of each annuity payment.

Moreover, the rules allow ρ_x to remain the same, even beyond the annuitant's life expectancy. This means that even though the principal should be considered completely returned and the subsequent payments should wholly be considered taxable income, the annuitant will still be taxed on only part of them.

Now, recall from the preceding discussion that tax arbitrage will occur if the tax payment under a mortality swap is lower than under a bank deposit; i.e.,

$$\frac{\tau\rho_x}{a_x(r)} < \tau r,$$

or

$$\tau\left(r - \frac{\rho_x}{a_x(r)}\right) > 0.$$

Based on the interpretation of $\rho_x / a_x(r)$ as the taxable income, this condition simply states that tax arbitrage will occur if the tax rules assign "too-low" taxable income from the annuity to each tax year. When an exponential distribution is used to price annuities, arbitrage does not occur because the annuity price, $a_x(r)$, is such that $\rho_x / a_x(r)$ is equal to r. That is, the taxable income assigned by the tax rules happens to be appropriate for the mortality swap.[22] However, when a distribution from the EMTT class is used, $a_x(r)$ is higher than in the exponential case for a given life expectancy and a given level of interest rate. This causes the assigned taxable income to be too low because, as mentioned earlier, $\rho_x / a_x(r)$ is a decreasing function of $a_x(r)$.

$a_x(r)$ under an EMTT distribution is higher than its exponential counterpart for the following reason. Under an EMTT distribution, the probability of survival during the early years is higher than under an exponential distribution. Therefore, the contributions of deceased annuitants to the annuity pool during the early years will be lower under an EMTT distribution. This means that in these early years, insurance companies have to rely more on interest income to meet their payment obligation. For a given level of interest rate, the only way to generate sufficient interest income is to start with a sufficiently high principal. This results in a higher value for $a_x(r)$.

Mortality Swap in the U.S. Markets

Under the U.S. tax rules, for the vast majority of life insurance policies, their death benefits are not taxable. Also, insurance premiums are not tax-deductible. Therefore, the tax treatment for the insurance side of a mortality swap is the same as in the Canadian case above.

However, for annuities, the U.S. Internal Revenue Service (IRS) specifies that the taxable portion, ρ_x, be increased to one (i.e., 100 percent) after the annuitant's life expect-

[22] Note that the authors do not claim that this assigned amount is appropriate for the annuity when it is held by itself.

ancy has been reached.[23] Under such rules, a decline in after-tax income will occur when the annuitant exceeds his or her life expectancy. As a result, the expected effective tax rate will be higher. The IRS adopted this practice in 1986. Before that time (from 1954 to 1986), the U.S. taxed annuities in the same manner as the current Canadian method (i.e., the exclusion ratio remained constant during the annuitant's life).

As a result of the difference in the way annuities are taxed in the U.S., a mortality swap no longer replicates a bank deposit on an after-tax basis, and the comparison technique that the authors use in the Canadian case no longer applies. One can alternatively compare the present values of the two investment choices to determine whether tax arbitrage exists. That determination will, however, depend on one's (subjective) choice of mortality distribution used in the calculation. Also, if tax arbitrage exists, it will do so only in an "expected" sense. It is possible that *a posteriori*, a mortality swap turns out to be worse than a bank deposit. This will happen if the individual *significantly* outlives his or her life expectancy as specified by the tax authorities, $E[T_x^g]$. In other words, under the U.S. rules, a mortality swap cannot completely remove the mortality risk associated with annuities and insurance policies. Therefore, the authors believe that it is impossible that a sure tax-arbitrage opportunity will exist under the U.S. tax rules.

Recently, Brown, Mitchel, Poterba, and Warshawsky (1999) proposed an alternative scheme that is similar to the current Canadian scheme (i.e., with a constant ρ_x) while preserving the higher expected effective tax rate. Their proposed scheme is intended to prevent a significant drop in annuitants' income after they reach their life expectancies. They show that the current U.S. tax rules can result in a decline of as much as 60 percent in after-tax annuity income in real term. To preserve the higher effective tax rate, they determine ρ_x such that it provides the same expected present value of tax payments as the expected present value under the current rules.[24]

The proposed scheme will allow a mortality swap to again replicate a bank deposit, and thus completely remove the mortality risk. Nevertheless, because their proposed ρ_x will be at a higher level than in Canada, the present authors believe that tax arbitrage will still be unlikely, though less so than under the current scheme.

There is another reason that tax arbitrage from a mortality swap is unlikely in the U.S. market. U.S. courts have in the past issued rulings on "combination plans" into which an insurance company packaged an annuity and an insurance policy in a manner not significantly different from the mortality swap discussed here. Under the

[23] To be precise, the IRS specifies an "exclusion" ratio, which is $1 - \rho_x$. Therefore, this means that the exclusion ratio will decline to zero after the life expectancy has been reached. See Brown, Mitchell, Poterba, and Warshawsky (1999) for a discussion of the U.S. regulations.

[24] Therefore, their proposed method is different from the one the U.S. used from 1954 to 1986. Under their method, they first calculate the expected present value of taxes that an annuitant has to pay under the current rules (under which the taxable portion increases to 1 after the annuitant reaches his or her life expectancy). Based on that amount, they then come up with a new ρ_x that does not change through time but yields the same expected present value of taxes. On the contrary, the old U.S. rules, like the current Canadian rules, determined ρ_x based on the annuitant's life expectancy and the annuity's price and payments.

rulings, the proceeds from the insurance policy would be subject to income tax (in the hands of the beneficiary) to the extent that the proceeds exceed the net premiums that would have to be paid for the insurance policy if it were a standalone policy.[25] Assuming that the rulings also apply to the authors' case, the after-tax payoffs of a mortality swap will be further reduced, and thus tax arbitrage from it will be even more unlikely.

Mortality Swaps Under Stochastic Interest Rates

The authors' analysis so far has been done under the assumption of a flat term structure of interest rates. With a non-flat term structure and stochastic interest rates, the comparison between a mortality swap and a risk-free bank deposit has to be done in a different manner. This is because the payoff from the deposit will vary from year to year. As a result, the existence of tax arbitrage should be determined by comparing the expected after-tax present values of the two investments where the payoff of the bank deposit for any future year is based on the one-year forward interest rate for that year.[26] While this determination will depend on one's (subjective) choice of mortality distribution used in the calculation, the preceding numerical analysis suggests that, *a priori*, a mortality swap will still be a better investment choice than a bank deposit. This is because the swap's equivalent pre-tax returns (in Table 1) are generally higher than the interest rates for all points in the term structure. Note also that this tax arbitrage exists only in an "expected" sense. It is possible that, *a posteriori*, a mortality swap turns out to be worse than a bank deposit. This will happen if a dramatic rise in interest rates occurs shortly after the swap was constructed. A perfect analogy to this is the case of interest-rate swaps whose current values are positive. The current positive values are the result of the current term structure of interest rates. Because interest rates can move in an adverse direction in the future, it is not certain that the positive values will be realized at the maturity of the swaps.

The Effects of Market Frictions

Except for income taxation, the authors have so far assumed away other market frictions. In this section, the authors investigate the effects of transaction costs and asymmetry of information on the arbitrage opportunities involving mortality swaps.

Asymmetry of Information and Loading

To protect themselves from the problem of adverse selection, insurance companies use mortality distributions that are different from that of the population as a whole. When pricing an annuity, they assume higher probabilities of survival than the population average. In other words, they assume that their annuitants will live longer than average. However, they assume lower-than-average probabilities of survival (i.e., their policyholders have a shorter life expectancy than average) when they price

[25] See Helvering v. LeGierse, 312 U.S. 531 (1941) and Kess *v. U.S.*, 26 AFTR 2d 70-5839 (S.D. Ohio 1970).

[26] Because the discounting is done using the term structure of after-tax interest rates, the present value of the bank deposit is equal to the initial investment. Therefore, this comparison amounts to determining whether the present value of the payoffs a mortality swap is greater than the initial investment.

an insurance contract. This practice causes both the annuity price and the insurance premium to be higher than they would be had the population-average probabilities been used. Insurance companies must also pay commissions and other expenses and make a profit. This further raises the price and the premium.

Let l_a represent the proportional loading costs associated with the annuity, expressed as a percentage of the annuity's frictionless price, $a_x(r)$. Similarly, let l_i represent the friction costs associated with the insurance policy, expressed as a percentage of the frictionless premium, $i_x(r)$. Then the annuity's price in the presence of frictions becomes:

$$(1 + l_a)\ a_x(r) \qquad l_a > 0, \tag{8}$$

while the insurance premium becomes:

$$(1 + l_i)\ i_x(r) \qquad l_i > 0. \tag{9}$$

To see the effect of l_a and l_i on the opportunity for tax arbitrage, consider again an individual who is confronted with a choice between a \$1 investment in a mortality swap and the same amount of investment in a risk-free bank deposit. The after-tax payoffs from the two alternatives are as follows:

Investment's After-Tax Payoff	State of Nature	
	While Alive	At Death
Mortality Swap	$\$\frac{1-\tau\rho_x}{(1+l_a)a_x(r)} - \$(1+l_i)i_x(r)$ per year	\$1
Bank Deposit	\$ (1 - τ) r per year	\$1

where the authors assume that there is no cost associated with the bank deposit.

In this case, the tax arbitrage condition becomes:

$$\$\frac{1-\tau\rho_x}{(1+l_a)a_x(r)} - \$(1+l_i)i_x(r) > (1-\tau)r, \tag{10}$$

which can be rewritten in terms of ρ_x as:

$$\tau\rho_x < [\tau + \tau l_a - l_a]ra_x(r) - [l_i + l_a + l_i l_a]a_x(r)i_x(r). \tag{11}$$

By comparing Inequality (11) to Inequality (6),

$$\rho < ra_x(r) \text{ or } \tau\rho < \tau ra_x(r),$$

one can see that in the presence of frictions, the taxable portion, ρ_x, has to be smaller than before for arbitrage to exist. In other words, the portion of annuity income exempted from taxes must be greater than before. Mathematically, this is because (1) $[\tau + \tau l_a - l_a]$ is less than t by the fact that $0 < t < 1$ and $l_a > 0$; and (2) $[l_i + l_a + l_i l_a]a_x(r)i_x(r)$ is positive by the fact that l_i, l_a, $a_x(r)$, and $i_x(r)$ are all positive. Intuitively, this is because \$1 now buys a smaller stream of pre-tax annuity income that then has to be used to pay taxes and finance a higher insurance premium in a mortality swap. As a result, the swap's return is reduced, and tax arbitrage might not exist in the presence of frictions.

Unlike the frictionless (with taxes) case, the existence of arbitrage depends on the individual's marginal tax rate. The higher the marginal tax rate, τ, the easier it is for Inequality (10) to be satisfied. The intuition for this is that the effect of taxes on the payoffs of a mortality swap is less than on a bank deposit because only a portion of the annuity income is taxable. This suggests that, everything else being equal, individuals in high tax brackets have a better chance of benefiting from a mortality swap.

While the logic of Theorems 3 and 4 still applies in this case, the existence of arbitrage is, more practically, an empirical question.

CONCLUSION

In this article, the authors document and explain the existence of a tax arbitrage opportunity involving a mortality swap. The authors label the transaction a mortality swap because the investor takes on mortality risk by acquiring an immediate life annuity and then swaps it back by purchasing life insurance. The authors show that on a before-tax basis, investors will be indifferent between a mortality swap and a bank deposit that it replicates. However, in the presence of the Canadian tax rules and under realistic mortality distributions that are used to price annuities and insurance, a mortality swap will provide a higher after-tax rate of return than that of a generic bank deposit. This tax arbitrage opportunity is the result of the way annuity income is taxed in Canada and, to a certain extent, the fact that the inside buildup in a life insurance policy accumulates tax-free. Under the Canadian rules, the taxable portion of annuity income is too low and remains constant at that level throughout annuitants' lives. This allows a mortality swap to replicate the payoff of a bank deposit on a before-tax basis and exceed it on the after-tax basis.

The authors use observed annuity prices and insurance premiums to estimate the magnitude of the excess return for individuals of various ages and tax rates. The discrepancy in the rates of return indeed exists. The results show that the older the individuals are and/or the higher their tax rates are, the more they stand to gain from this strategy.

The authors discuss the reasons that this tax arbitrage is less likely under U.S. tax rules. Under the U.S. rules, the taxable portion increases to 100 percent after annuitants reach their life expectancies. In this case, a mortality swap can no longer replicate a bank deposit. As a result, the existence of tax arbitrage depends on each individual's subjective probability of survival. The authors believe that tax arbitrage is unlikely under the U.S. rules because the increase in the taxable portion results in a higher effective tax rate for annuities. This is consistent with the results of Brown et al. (1999), which show small differences between before- and after-tax expected values of life annuities in the U.S. market. In addition, past court rulings in the U.S. markets on the tax status of a comparable product called "hedged insurance policies" indicate that the proceeds from the insurance portion of the swap may be taxable. If this is the case, the possibility of tax arbitrage is remote.

Past studies on the effects of income taxes on security prices [see, for example, Dybvig and Ross (1986) and Dammon and Green (1987)] showed that equilibrium will not exist unless the investors are prevented from taking advantage of a price (or return) discrepancy by short-sale restrictions, progressive marginal tax rates, or some other form of friction. In the case of mortality swaps, however, the return discrepancy can persist in the Canadian markets for mainly two reasons. First, a mortality swap is not scalable because individuals are limited by the amount of insurance that they can purchase. Second, a mortality swap is not entirely riskless, as tax laws might change without providing a "grandparent" clause, in which case the swap's after-tax return may turn out to be lower than that of a bank deposit.

APPENDIX A

The authors obtained Canadian insurance and annuity quotes as of July 1, 1999, for female individuals who are between 50 and 80 years old, in five-year increments. The annuity quotes were provided by CANNEX Financial Exchanges Limited, and the insurance quotes were provided by SunLife and Transamerica.[27]

The life annuity prices were quoted in terms of the monthly payments that the buyer will receive from the annuities, assuming a $100,000 purchase on July 1, 1999, with the first annuity payment on August 1, 1999. The annuity quotes were from contracts that have no guarantee periods and are based on a single individual's life.[28]

The CANNEX system also provides the taxable amount of each annuity payment, which is based on CCRA's method of computing ρ_x. Interestingly, the 10 to 15 most competitive annuity quotes as of July 1, 1999, showed a wide variation that can range from 1 percent to 15 percent of the best rate in the group. This variation could not be attributed to credit quality alone. Casual discussions with pricing actuaries indicated that sometimes insurance companies deliberately propagate noncompetitive annuity prices in order to discourage the inflow of funds. For this reason, the authors decided to use the average of the five best rates in the group as their proxy for annuity prices. In any event, because of the collection and distribution of the quotes by CANNEX, the life annuity market in Canada seems to have been commoditized and is quite transparent. The annuity side of a mortality swap is therefore easy to price.

The same cannot be said about the life insurance market. The myriad of life insurance products (such as term, whole life, and universal) do not allow for one standard comparison across companies and rates. Health status complicates matters by creating a collection of rates based on various underwriting criteria. For the purposes of a mortality swap, the authors chose a "term-to-100" insurance policy, which is a policy that maintains constant level of premiums until age 100. At age 100, the policy is

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[27] CANNEX is a financial data intermediary that collects life annuity quotes and quotes of other products from all major Canadian insurance companies. CANNEX then provides an online report that summarizes the 10 to 15 most competitive life annuity quotes available for a particular sex, age, and investment combination. The annuity quotes are listed together with the names of the issuers and their credit ratings. These quotes are statistically reliable and usually guaranteed for a period of 15 to 30 days.

[28] In other words, the quotes are from contracts that do not allow co-annuitants.

considered paid in full and no further premiums are required. A term-to-100 policy does not have a cash value and provides coverage for life even if the insured survives beyond age 100. As mentioned earlier, this product is the closest policy to the insurance side of a mortality swap.

The authors obtained quotes from SunLife and Transamerica for term-to-100 policies for female individuals of various ages. These quotes were applicable only to those with the highest quality health risk (i.e., preferred rates) and would probably be conditional on the insured's passing a medical examination. The insurance premiums were quoted in terms of monthly rates based on a \$100,000 constant death benefit. The policies became effective on July 1, 1999, but would require a 30-day standard approval period. On August 1, 1999, the first (monthly) premium would be due and payable, conditional on passing the medical exams. The first (monthly) annuity income would therefore coincide with the first (monthly) insurance payment.

Because the annuity income was quoted on a monthly basis, the authors annualized the quotes by multiplying them by 12. The annualized insurance premiums divided by \$100,000 correspond to $i_x(r)$ in our notation. Likewise, \$100,000 divided by the annual payment from the annuity corresponds to $a_x(r)$.

APPENDIX B

Proof that annuity prices under the EMTT class of distributions are higher than the exponential prices, given that $E[T_x^g] = E[T_x^c] = E[T]$.

Let $p_e(x,s)$ be the probability that an x-year-old individual will still be alive s years from now under the exponential distribution whose first moment (i.e., life expectancy) is $E[T]$. Also, let $p_a(x,s)$ be the probability that an x-year-old individual will still be alive s years from now under a distribution from the EMTT class *with the same life expectancy*. Based on the equation for annuity prices in (1), the authors will show that:

$$\int_0^\infty e^{-rs} p_a(x,s)ds > \int_0^\infty e^{-rs} p_e(x,s)ds.$$

First, the authors note that life expectancy is simply the integration of the probability of survival over time horizons of zero to infinity. Because life expectancy is the same under both distributions,

$$\int_0^\infty p_a(x,s)ds = \int_0^\infty p_e(x,s)ds. \qquad (12)$$

Now, based on the authors' definition of the EMTT class, any mortality distribution in this class has the following property:

- There exists some age y; $y > x$, such that (a) the probability of survival up to *any* age before y under the distribution is higher than its exponential counterpart; and (b) the probability of survival up to *any* age beyond y under the distribution is lower than its exponential counterpart; i.e., $p_a(x,s) > p_e(x,s)$ for all $s \le y - x$, and $p_a(x,s) < p_e(x,s)$ for all $s > y - x$.

By the additive property of the integration, Equation (12) can be rewritten as:

$$\int_0^{y-x} p_a(x,s)ds + \int_{y-x}^{\infty} p_a(x,s)ds = \int_0^{y-x} p_e(x,s)ds + \int_{y-x}^{\infty} p_e(x,s)ds.$$

Equivalently,

$$\int_0^{y-x} p_a(x,s)ds - \int_0^{y-x} p_e(x,s)ds = \int_{y-x}^{\infty} p_e(x,s)ds - \int_{y-x}^{\infty} p_a(x,s)ds$$

or

$$\int_0^{y-x} \left[p_a(x,s) - p_e(x,s)\right]ds = \int_{y-x}^{\infty} \left[p_e(x,s) - p_a(x,s)\right]ds.$$

Then, by the fact that e^{-rs} is decreasing in s, the authors have that:

$$\int_0^{y-x} e^{-rs}\left[p_a(x,s) - p_e(x,s)\right]ds > \int_{y-x}^{\infty} e^{-rs}\left[p_e(x,s) - p_a(x,s)\right]ds$$

or

$$\int_0^{y-x} e^{-rs}\left[p_a(x,s) - p_e(x,s)\right]ds + \int_{y-x}^{\infty} e^{-rs}\left[p_a(x,s) - p_e(x,s)\right]ds > 0,$$

which, by the additive property, can be rewritten as:

$$\int_0^{\infty} e^{-rs}\left[p_a(x,s) - p_e(x,s)\right]ds > 0$$

or

$$\int_0^{\infty} e^{-rs} p_a(x,s)ds > \int_0^{\infty} e^{-rs} p_e(x,s)ds.$$

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