Can Life Insurance be used to Hedge Payout Annuities?
Part 1: Modeling Longevity Risk

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Summer 2003

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"…Moody's believes that the two main risks to insurance companies from payout annuities are embedded equity guarantees and inaccurate longevity assumptions….

…Aggressive longevity assumptions relate to mortality risk assumed, or the risk that annuitants will on average live longer than originally anticipated by the insurer, thus extending the period for which the insurer is obligated to make these monthly payments….

Moody's Investors Services, May 2002

1. Introduction:

Insurance companies and re-insurers alike have expressed grave concern about guaranteeing mortality on the sale of immediate annuities. This is due to the longevity risk that medical discoveries might increase our lifespans beyond currently projected mortality tables, perhaps leaving the insurance company with infinitely lived annuitants. Many companies go so far as to impose an explicit mortality risk charge, on a perpetual asset basis, to cover this contingency.

The traditional economic response that in-force life insurance might serve as a hedge against this (diversifiable) risk is dismissed by the industry since the target group for the policies are distinct. Immediate annuities are sold to the old, while life insurance is purchased by the young. They claim that an increase in population longevity will adversely impact the liabilities of the former, but marginally impact the profitability of the latter. Furthermore, they argue that the duration and especially the lapsation behaviour of the opposing liabilities are mismatched and hence can not properly hedge each other.

Our research agenda in this paper is to investigate the impact of unexpected longevity risk embedded within payout annuity contracts, on the profitability and return-on-equity from this business.

On a purely theoretical level, life insurance and annuity liabilities are sensitive in opposing directions to changes in the entire mortality table, albeit in different magnitudes. Therefore, our continuing objective is to use insights from financial derivative pricing to investigate the extent to which life insurance can serve as a *dynamic hedge strategy* – from an Asset Liability Management perspective – against unexpected mortality risk.

On a technical level, we capture the longevity risks by modelling the instantaneous hazard rates – a.k.a. the force of mortality -- using a diffusion process, as opposed to a deterministic actuarial curve. This framework allows us to model surprises in the mortality curve, while maintaining a coherent pricing basis.

The remainder of this paper is organized as follow. In section 2 we introduce the concept of diffusion hazard-mortality rates and show how to derive the value of life-contingent claims under this model. Section 3 proceeds to model the impact of un-expected mortality improvements on the profitability of payout annuities. Section 4 provides numerical examples and Section 5 concludes the paper.

Overall, our main qualitative results are as follows. First, a relatively small change in the hazard-mortality rate curve can have a substantial impact on the return-on-equity (ROE) and internal rate of return (IRR) of a leveraged block of annuity business. For example, an *ex-ante* target IRR of 15% can become an ex-post IRR of (minus) -20% under an

unexpected, but modest, reduction of 30% in retirement hazard-mortality rates. However, more interestingly, the sensitivity of profitability to changes in hazard-mortality rates is much more severe at higher ages. In other words, misestimating mortality on an annuity that is issued at age 75, is much more devastating than the same level of mis-estimation on a life annuity issued at age 55. We will elaborate on the reasons for this phenomenon later in the paper. The practical implication of this statement, is that the payout annuity risk profile might induce a risk premium in the annuity 'term structure' that will be investigated empirically in future research.

Finally, an additional follow-up paper will perform a similar mortality sensitivity analysis on life insurance contracts. We will then proceed to develop a methodology for computing the required death benefit of a whole life policy that is sold to a **y**-year old, in order to hedge $1 of annuity premium sold to an **x**-year old, for any given **y** and **x**.

2. Diffusion Hazard-Mortality Rates.

In this section we propose a slightly different way of thinking about payout annuity pricing. Traditional actuarial pricing assumes a force-of-mortality curve, and a force of interest curve, and then discounts cash-flows to arrive at actuarial present values.

Our approach is as follows. We start by defining a diffusion hazard-mortality rate process:

$$dh_{x+t} = v(h_{x+t})dt + \sigma(h_{x+t})dB^h{}_t$$

where the v and σ functions capture the drift and diffusion of the hazard-mortality rate. The conditional survival probability becomes:

$$(_t p_x) = \exp\{-\int_0^t h_{x+s}ds\}$$

Finally, if we assume a diffusion process for the pricing interest rate, the annuity factor becomes:

$$a_x = \int_0^\infty E[\exp\{-\int_0^t (r_s + h_{x+s})ds\}]dt$$

The annuity factor is obtained by integrating <u>both</u> the uncertain interest rate and the uncertain hazard rate against the appropriate payoff function. The paper by Milevsky and Promislow (2001) provides some parametric specification for the hazard-mortality rate function and illustrates how this methodology can be applied to price contingent claims that depend on future (stochastic) annuity prices.

For now, our objective is to simply set a framework for describing the evolution uncertain hazard rates, the next section will provide some simple numerical examples involving a constant 'shift' in the hazard-mortality curve.

3. The Impact of Longevity Risk

Despite the overwhelming benefits of a longer life, at first glance, the implications of increased longevity are quite negative for any book of annuity business. The issue conjures up an image of annuitants living to age 150 and beyond, and insurance companies on the verge of insolvency.

Clearly, the impact of such longevity risk depends on the exact timing and magnitude of the scientific and medical breakthrough, as well as a better understanding of how annuities are priced and valued.

To that end, we review some basic actuarial pricing and reserving theory, in order to gauge the impact of un-anticipated longevity improvements on insurance company profitability and solvency.

On a basic level, one can represent the price of -- or the insurance liability created by -- a $1-for-life annuity in the following manner:

$$a_x = f(\mathbf{x}, \mathbf{q}, \mathbf{l}, \mathbf{r}, \mathbf{s})$$

In the above expression, **x** is the age at which the annuity is issued, **l** captures the expense loading, the vector **q** represents the mortality table, the vector **r** represents a term structure (yield curve) of interest rates, and the most critical variable **s**, is the profit spread. One can think of **s** as the difference between what the insurance company will earn on it's assets, and what it 'credits' the annuitant, net of expenses. Intuitively, the annuity factor is decreasing in x, and r, but increasing in l and s. In other words, older people pay less, and annuity factors are reduced in a higher interest rate environment. But, greater expenses and profit spreads will increase the price per dollar.

For example, in a flat **r** = 6% yield-curve pricing model, one might see a profit spread on the order of **s** = 1%, and a proportional expense loading of **l** = 10%. In this simplified case, the annuity factor for the price of (or the insurance liability created by) a $1-for-life annuity would be:

$$a_x = (1+0.1)\sum_{i=1}^{\infty} \frac{(_i p_x)}{(1+0.06-0.01)^i} ,$$

where the numerator is the well-known conditional probability of survival.

More precisely, if we use the Society of Actuaries 1994 Group Annuity Mortality (static, unisex) Table, the actual annuity factors would be 15.69, 13.73 and 11.11, for ages 55, 62 and 70 respectively. Naturally, the younger the issue age of the annuitant, the more they must pay (and the greater the required reserves) for the same $1-for-life guarantee.

In practice, the annuitant acquires a single premium immediate annuity (SPIA) with an initial sum of **W**, thus guaranteeing a life-annuity of **W/ax** for life. For example, a 55 year old with $100,000 would be entitled to an annual income of $100,000/15.69 = $6,373 for life. In the event of a period certain guarantee, the annuity factors would be higher -- since the mortality rates in the numerator would be set to a value of one during the guarantee period – and thus the annual income would be reduced in proportion to the length of the guarantee.

This is, roughly speaking, how the pricing is determined. In practice, of course, the valuation rate would be applied in the denominator to determine the required reserves, while the actual pricing would more closely resemble the above. However, for our purposes, we deliberately blur the distinction between pricing and valuation since we are interested in the broader impact of unanticipated longevity risk. For now, we imagine that every dollar of premium must be placed in reserves, but no more. Thus ignoring capital issues and any possible surplus strain created by statutory valuation rates. The gap between the two will not change the main argument.

To gauge the impact of mortality improvements, imagine a situation in which a life annuity is issued and priced at age 62, with a 100 basis point profit margin, assuming the SoA 1994 GAM (static, unisex) table captures the underlying population. As one can see from Table #1, the life expectancy – at the issue age – is 83.8, and the *ex ante* profit spread is 100 basis points.

If, however, the insurance company overestimated the <u>true</u> force of mortality of the group as a result of unexpected mortality improvement, the *ex post* profit spread will clearly be lower than 100 basis points, the question is 'by how much?'.

Let us look a bit more closely at the precise causes of death.

At the advanced ages, <u>approximately </u>10% of deaths can be attributed to strokes together with pneumonia, an additional 30% can be attributed to cancer and diabetes, 40% is due to heart disease, and the remaining 20% are accidents, suicide and formally classified as others. We group these factors together fully cognizant of their somewhat unrelated medical factors. Likewise, the exact fraction will depend on the population in questions, their sex and age at death. For now, we <u>assume</u> the fractions are constant.

Now, imagine that science finds a cure to all strokes and pneumonia. In this case, the (hazard rate) force of mortality would be reduced at each age by a factor of 10%. If we cure cancer and diabetes, mortality would be reduced by 40%, and if we can eliminate heart decease, mortality would be reduced by 80%. Of course, we are making a very critical and implicit assumption that eliminating one particular cause of death will have no impact on the other competing sources of death. In other words, we are assuming that all causes are statistically independent and can be 'additively' removed from the hazard-mortality rate. In practice, this will not be the case, and it's hard to imagine the hazard-mortality rates changing in such a predictable fashion. Nevertheless, our

objective is gauge the impact of a fractional reduction in the hazard-mortality rate, as opposed identifying the exact source and cause of this reduction. That task we will leave to the biostatisticians and the medical profession.

On a technical level, the revised force of mortality would be related to the (assumed, pricing) force of mortality via the relationship:

$$\mu'_x = (1+f)\mu_x,$$

where **f** < 0 represents the fractional reduction in mortality.

When we reduce mortality, each and every q_x rate in the appropriate cohort table used to price the annuity is reduced by 10%, 40%, and 80% respectively. For example, under an *f = −10%* shock immediately upon issuing the annuity, the modified cohort probability of a 55-year-old surviving to age 59, would be approximately:

$(1-(0.9)q_{55})$ x $(1-(0.9)q_{56})$ x $(1-(0.9)q_{57})$ x $(1-(0.9)q_{58})$.

Thus, at each age, a *fixed fraction* of deaths are eliminated as a proxy for the reduction in various decrements. In practice, of course, a properly detailed methodology would involve reducing each and every q_x by the fraction of deaths caused by any particular factor. We use the word approximately above, because the actual mortality adjustment would have to take account of fractional age payment by perturbing the instantaneous hazard-mortality rate, as opposed to the q_x values themselves.

We stress once again that we are approximating reality somewhat by assuming that a constant fraction **f** of deaths for any given age can be attributed to a specific illness, as opposed to an age related fraction **fx**. In practice, the number would vary. But, for our purposes we are interested in the effect of mortality improvements, as opposed to the reasons, per se.

Furthermore, assuming the improvement (medical breakthrough) occurs immediately after the annuity is issued (sold, priced), the true annuity factor should have been:

$$a'_x = f(\mathbf{x}, \mathbf{q'}, \mathbf{l}, \mathbf{r}, \mathbf{s}),$$

where the prime symbol above the **q** denotes the true mortality vector. *Ceteris paribus*, for any given **x, l, r** and **s**, the true annuity factor should have been <u>higher </u>for any given decline in the mortality rates.

The final step is to invert and solve for the profit spread that equates the original (lower) annuity factor -- that was originally used to price the annuity – and the true (higher) annuity factor.

$$\max \quad s'$$
$$s.t. \quad f(\mathbf{x},\mathbf{q},\mathbf{l},\mathbf{r},\mathbf{s}) = f(\mathbf{x},\mathbf{q}',\mathbf{l},\mathbf{r},\mathbf{s}')$$

Mathematically, we are solving for the largest profit spread that equates the two annuity factors. The question we answer is: What will the profit spread actually be, given what they originally charged the annuitant? Naturally, for any given level of mortality improvement, $\mathbf{s}' < \mathbf{s}$, and if the improvement is large enough (i.e. $\mathbf{f} << 0$), the implied spread might be negative.

Table #1

Single Premium Immediate Annuity Issue Age

Mortality Reduction	Unisex 55		Unisex 62		Unisex 70	
	Life Exp.	*Spread*	*Life Exp.*	*Spread*	*Life Exp.*	*Spread*
0%	82.9	+100 bp	83.8	+100 bp	85.6	+100 bp
-10%	83.8	+ 85 bp	84.7	+ 77 bp	86.4	+ 60 bp
-40%	87.4	+ 39 bp	88.1	+ 4 bp	89.4	- 67 bp
-80%	97.7	- 36 bp	97.9	- 111 bp	98.6	- 257 bp

The table illustrates the impact of an unexpected improvement in life expectancy, driven by a constant proportional reduction in the (hazard rate) force-of-mortality. These ratios roughly coincide with the average causes of death listed on the left.

Pricing and reserves are based on Gompertz approximation to 1994 GAM (static) table, 10% expense loading and a 6% (minus the profit spread) flat discounting.

For example, if annuity is issued at age 62, and subsequently mortality declines by 40%, the book of business will earn only 4 bp, as opposed to the 100 bp used for pricing.

4. Numerical Examples

Table #1 provides some numerical results assuming a particular mortality table and interest rate curve. What we find most interesting is that the higher the issues age, the greater the impact (on profitability) of a given percentage improvement in mortality. For example, reducing the hazard-mortality rate by 40% in Table #1, will still leave the insurer with a profit spread of +39 basis points, at issue age 55, but a –67 bp spread at issue age 70. One can do the same exercise with an individual annuity mortality table, such as the IAM or with some form of dynamic projection and obtain results on the same order of magnitude.

The intuition for this result is as follows. The younger the at-issue age of the annuity, the closer the liability is to a perpetuity obligation, and the less important mortality improvements become. Stated differently, the annuity issued at age 55 – with a factor of 15.69 per dollar -- could be paid in perpetuity, assuming a 6.37% interest rate. (Note:

1/15.69 = 6.37%). In some sense, if the insurance company can invest the assets so as to earn 37 basis points more than the assumed spread, their risk profile is completely immune to mortality improvements. In contrast, the annuity issued at age 70 – with a factor of 11.11 per dollar – could only be paid in perpetuity assuming a 9% interest rate, which is 200 basis points greater than the assumed spread, and is much less likely to happen. At the extreme age of 80, the perpetuity rate would be 13%.

Table #2 **Single Premium Immediate Annuity Issue Age**
Analysis of ex post Internal Rate of Return (IRR)

Mortality Reduction	Unisex 55		Unisex 62		Unisex 70	
	Life Exp.	*Spread*	*Life Exp.*	*Spread*	*Life Exp.*	*Spread*
0%	82.90	15.0%	83.80	15.0%	85.60	15.0%
-10%	83.80	12.1%	84.70	10.3%	86.40	6.7%
-40%	87.40	2.6%	88.10	-4.8%	89.40	-20.1%
-80%	97.70	-12.8%	97.90	-28.0%	98.60	-60.2%

It is somewhat counterintuitive, but our model seems to indicate that the risk profile of the payout annuity business is greatly reduced the younger the issue age. For example, a life annuity purchased by a 20-year old is essentially priced like a perpetuity, and thus mortality improvements that occur (immediately) after issue, do not impact the risk factor by much.

We can perform the same analysis on Internal Rate of Return values. For the purposes of this calculation, we abstract from reality by assuming a 5% target surplus, and a flat 6% interest rate environment. We assume the business is initially priced to yield a 15% (pre-tax) IRR. We then 'shock' mortality and solve for the 'new' IRR assuming that the cash-flow stream is based on the new mortality rates. Table #2 provides numerical results.

Thus, an annuity issued at age 55 that is priced to yield a (pre-tax) IRR of 15%, would result in an *ex post* IRR of 12.1%, if *realized* mortality-hazard rates are better (worse, from the insurance company's point of view) than *assumed* mortality by 10%. Furthermore, if realized hazard-mortality rates are better than the assumed curve by 40%, the IRR for the age 55 at-issue book of business will drop to 2.6%.

Finally, notice also that despite an 80% reduction in the mortality rates – which we loosely describe as a virtual elimination of cancer, stroke, pneumonia and heart disease -- the revised life expectancy at age 70 is slightly less than 100, compared to the original 85.6. While we are not dismissing a 15 year increase in life expectancy, we find

it interesting that an 80% reduction in the death rate for any given age, <u>only</u> adds 15 years. Although we are far from demography or actuarial experts, from a purely mathematical point of view, a 62 (unisex) year-old annuitant with a current life expectancy of age 83.8, would have to experience a 98% reduction in the force of mortality at all future ages, to expect to live to the biblical 120 upper bound.

5. Conclusion:

In this paper we have investigated the impact of unexpected improvements in human longevity on the profitability of a book of payout annuities. Our approach is to assume that the pricing hazard (force of mortality) rate changes after the annuity is issued, which allows us to invert the new annuity factor to obtain the *ex post* profit spread and return-on-equity. In the process, we have introduced the concept of a diffusion hazard-mortality rate, which differs from the traditional approach to pricing payout annuities.

Our main result is that (1.) profitability is quite sensitive to changes in mortality rates, and (2.) the younger the at-issue age of the payout annuity, the less sensitive the book of business is to un-expected changes in mortality. This phenomenon is due to the fact that the annuity factor is 'closer' to perpetuity – thus being invariant to mortality risk – the younger the annuitant.

Further research will use these results to develop effective dynamic hedges for 'longevity risk' by locating suitable -- term and whole life -- insurance contracts with an exact opposite exposure to the risk.

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